Differential Decoder for MAC Based Two-User Communication Systems

Manav R. Bhatnagar, Member, IEEE and Are Hjørungnes, Senior Member, IEEE

Abstract

We derive a decoder for differential data of two multiple access channel (MAC) based uplink users. Both users utilize differential modulation for transmitting their data. It is assumed that the users cannot exchange their information. In addition, it is assumed that the users transmit data over multiple access uplink channels simultaneously in the same frequency band and without any orthogonal signatures (no CDMA). Their transmissions are assumed to be perfectly synchronized in time, phase, and frequency. The decoder is obtained by maximizing the joint probability distribution function (p.d.f.) of consecutively received data samples. We first obtain a partially differential decoder which avoids the knowledge of channel of one user at the base-station. Then we derive a heuristic differential decoder which completely avoids the knowledge of the channels of both users. For the heuristic differential decoder to work properly, both users must use rotated constellations. Optimized values of rotation angles for different $M$-PSK constellations are numerically obtained. It is further shown by simulations that the proposed differential decoders perform better than a same rate existing TDMA based two-user transmission scheme.

I. INTRODUCTION

Differential modulation avoids the need of channel estimation at the receiver [1], [2]. In differential modulation, currently transmitted data consists of information of all previously transmitted data. This memory can be utilized by a maximum likelihood decoder for decoding of the transmitted data without knowing the channel gains. Differential modulation for single-user wireless communication has been extensively explored in [1], [2]. It is shown in the literature [3] that by introducing orthogonality among...
the transmissions of multiple users through TDMA, FDMA, or CDMA, the differential modulation can be implemented in a multi-user scenario. However, these coordinated transmission schemes reduce the effective data rate or number of users which can be accommodated in the multi-user system. In order to improve the data rate of a multi-user system, we need to consider transmissions over a general multiple access channel (MAC) [4], where the users transmit their information in a non-orthogonal manner, i.e., multiple users utilize the same time interval, same frequency band, and same signature waveform for their transmissions. Since the differential modulation avoids the need of training data transmission, a rate efficient multi-user system utilizes differential modulation. To the best of our knowledge, the problem of MAC based differential transmission by two-users utilizing same time interval, same frequency band, and same signature is not studied so far in the literature.

In this paper, we derive differential decoders for a MAC based two-user communication system where each user and the destination has a single antenna.

II. SYSTEM MODEL AND DIFFERENTIAL MODULATION

Consider a wireless communication system which consists of two users and one base-station (BS). Both users have their own data to transmit to the BS. It is assumed that the data transmitted by the users in time interval $n$ reaches the BS simultaneously with the same phase and frequency, i.e., their transmissions are perfectly synchronized in time, phase, and frequency. Moreover, both users utilize the same frequency band for their transmission and do not use any orthogonal signature (no CDMA). The users do not share their information. Therefore, both users independently transmit their symbols to the BS. Let $x_1[n]$ and $x_2[n]$ be the differential data of User 1 and User 2, respectively, to be transmitted in $n$-th time interval. Then the received data at the BS can be given as

$$y[n] = h_1 x_1[n] + h_2 x_2[n] + e[n],$$

(1)

where $h_1$ is the channel gain between the User 1 and the BS, $h_2$ is the channel gain between the User 2 and the BS, and $e[n]$ is the complex additive white Gaussian noise (AWGN) with zero mean and $\sigma^2$ variance. It is assumed that $h_1$ and $h_2$ are complex Gaussian random variables with zero mean and variance $\sigma_h^2$. Further, $h_1$ and $h_2$ remain constant over at least two consecutive time-intervals $n - 1$ and $n$. Encoding of $x_1[n]$ and $x_2[n]$ is explained in next subsection.
A. Differential Encoding

Let the symbol for User 1 be \( s_1[n] \in \Psi \), where \( |s_1[n]|^2 = 1 \) and \( \Psi \) is a unit-norm \( M \)-PSK constellation, and \( s_2[n] \in \Psi \), \( |s_2[n]|^2 = 1 \) be the symbol from User 2 to be transmitted in time interval \( n \). The differentially encoded data \( x_1[n] \) and \( x_2[n] \) are obtained as follows:

\[
\begin{align*}
    x_1[n] &= x_1[n-1]s_1[n], \\
    x_2[n] &= x_2[n-1]s_2[n].
\end{align*}
\]

(2)

In order to initiate the differential transmission, the initialization symbols \( x_1[0] = x_2[0] = 1 \) are transmitted by both users. Since \( |s_1[n]|^2 = |s_2[n]|^2 = 1 \), it follows that \( |x_1[n]|^2 = |x_2[n]|^2 = 1 \).

III. Decoding of Differential Data from Two-Users

A. Partially Differential Decoder Based on Two Consecutively Received Data Samples

Since \( e[n] \sim \mathcal{C}\mathcal{N}(0, \sigma^2) \), the conditional probability distribution function (p.d.f.) of \( y[n] \), given that \( h_1, h_2, x_1[n] \), and \( x_2[n] \) are known, can be written as

\[
f(y[n]|h_1, h_2, x_1[n], x_2[n]) = \frac{1}{\pi \sigma^2} \exp \left( -\frac{1}{\sigma^2} |y[n] - h_1 x_1[n] - h_2 x_2[n]|^2 \right).
\]

(3)

Let us now consider the data samples received in two consecutive time intervals \( n - 1 \) and \( n \) as \( y[n] = [y[n-1], y[n]] \). When \( h_1, h_2, x_1[n-1], x_2[n-1], s_1[n], \) and \( s_2[n] \) are known, then the conditional p.d.f. of \( y[n] \) can be expressed as

\[
f(y[n]|h_1, h_2, x_1[n-1], x_2[n-1], s_1[n], s_2[n]) = \frac{1}{\pi^2 \sigma^4} \exp \left( -\frac{1}{\sigma^2} \sum_{k=n-1}^{n} |y[k] - h_1 x_1[k] - h_2 x_2[k]|^2 \right).
\]

(4)

In order to find an estimate of the data \( s_1[n], s_2[n] \), the conditional p.d.f. of (4) is first maximized with respect to (w.r.t.) \( h_1, h_2, x_1[n-1], x_2[n-1] \), and, subsequently, over \( s_1[n], s_2[n] \), which results into minimization of the following ML metric:

\[
\Gamma[n] = \sum_{k=n-1}^{n} |y[k] - h_1 x_1[k] - h_2 x_2[k]|^2.
\]

(5)

From (2) and (5), the ML metric can be alternately written as

\[
\begin{align*}
\Gamma[n] &= |y[n-1] - h_1 x_1[n-1] - h_2 x_2[n-1]|^2 + |y[n] - h_1 x_1[n-1] s_1[n] - h_2 x_2[n-1] s_2[n]|^2, \\
&= \left| y[n-1] - \tilde{h}_1 - \tilde{h}_2 \right|^2 + \left| y[n] - \tilde{h}_1 s_1[n] - \tilde{h}_2 s_2[n] \right|^2,
\end{align*}
\]

(6)
where \( \tilde{h}_1 = h_1 x_1[n-1] \) and \( \tilde{h}_2 = h_2 x_2[n-1] \). By differentiating (6) w.r.t. \( \tilde{h}_1 \) and setting the result to zero, we get

\[
\hat{\tilde{h}}_1 = \frac{y[n-1] - \tilde{h}_2 + y[n] s_1^*[n] - \tilde{h}_2 s_2[n] s_1^*[n]}{1 + |s_1[n]|^2}.
\]  

(7)

Substituting the value of \( \hat{\tilde{h}}_1 \) of (7) into (6) we get the following ML metric:

\[
\Gamma[n] = \left| -y[n-1] s_1[n] + y[n] + \tilde{h}_2 (s_1[n] - s_2[n]) \right|^2
+ \left| y[n-1] |s_1[n]|^2 - y[n] s_1^*[n] + \tilde{h}_2 (s_2[n] s_1^*[n] - |s_1[n]|^2) \right|^2.
\]  

(8)

By using the fact that \( |s_i[n]|^2 = 1 \), where \( i \in \{1, 2\} \), (8) can be written as

\[
\Gamma[n] = 2 \left| y[n] - y[n-1] s_1[n] + \tilde{h}_2 (s_1[n] - s_2[n]) \right|^2.
\]  

(9)

Differentiating (9) w.r.t. \( \tilde{h}_2 \) and setting the result to zero, leads to the value of \( \tilde{h}_2 \) which minimizes the ML metric (5):

\[
\hat{\tilde{h}}_2 = \frac{y[n-1] s_1[n] - y[n]}{s_1[n] - s_2[n]}.
\]  

(10)

By substituting (10) in (9) it can be seen that \( \Gamma[n] = 0, \forall s_1 \in \mathcal{A} \), which indicates that the data of both users cannot be decoded differentially on the basis of two consecutively received data sample. However, (9) provides a decoder which can avoid the knowledge of the channel of User 1 and use only the knowledge of the channel of User 2 for decoding of the data of both users. This partially differential decoder is useful for scenarios where one user is at the cell boundary and its signals are very weak, hence, it is difficult to estimate its channel gains. Whereas the other user can be close to the BS and its channel gains can be estimated with small errors.

\[\text{B. Heuristic Decoder Based on Three Consecutively Received Data Samples}\]

In this subsection, we will develop a heuristic differential decoder based on three consecutively received data samples for the uplink two-user system. The data received in three consecutive time intervals \( n-2, n-1, n \) can be written as

\[
\begin{align*}
y[n-2] &= h_1 x_1[n-2] + h_2 x_2[n-2] + e[n-2], \\
y[n-1] &= h_1 x_1[n-2] s_1[n-1] + h_2 x_2[n-2] s_2[n-1] + e[n-1], \\
y[n] &= h_1 x_1[n-2] s_1[n-1] s_1[n] + h_2 x_2[n-2] s_2[n-1] s_2[n] + e[n].
\end{align*}
\]  

(11)
Assuming perfect knowledge of \( s_1[n-1] \) and \( s_1[n] \), we may rearrange the relations given in (11) as follows:

\[
y'[n-1] \triangleq y[n-1] - y[n-2]s_1[n-1] = h_2x_2[n-2] (s_2[n-1] - s_1[n-1]) + e'[n-1],
\]

\[
y'[n] \triangleq y[n] - y[n-1]s_1[n] = h_2x_2[n-2]s_2[n-1] (s_2[n] - s_1[n]) + e'[n],
\]

(12)

where the additive noise terms \( e'[n-1] = e[n-1] - e[n-2]s_1[n-1] \) and \( e'[n] = e[n] - e[n-1]s_1[n] \) are distributed as \( \mathcal{CN}(0,2\sigma^2) \). Moreover, the correlation matrix of \( e'[n-1] \) and \( e'[n] \) is given as

\[
A = \mathbb{E} \left\{ \begin{bmatrix} e'[n-1] \\ e'[n] \end{bmatrix} \begin{bmatrix} e'[n-1] \\ e'[n] \end{bmatrix}^H \right\} = \sigma^2 \begin{bmatrix} 2 & -s_1^*[n] \\ -s_1[n] & 2 \end{bmatrix}.
\]

(13)

From (12) and (13), the conditional p.d.f. of \( y'[n] \triangleq [y'[n-1], y'[n]] \) can be written as

\[
f (y'[n]|h_2, x_2[n-2], s_1[n-1], s_2[n-1], s_1[n], s_2[n]) = \frac{1}{\pi^2 \det(A)} \exp \left( - \left( y'[n] - \tilde{h}_2 [s[n-1], s_2[n-1]s[n]] \right) A^{-1} \left( y'[n] - \tilde{h}_2 [s[n-1], s_2[n-1]s[n]] \right)^H \right),
\]

(14)

where \( \det(\cdot) \) denotes the determinant and \( s[i] = s_2[i] - s_1[i], i = n-1, n \). In order to find an estimate of \( s_1[n-1], s_2[n-1], s_1[n], \) and \( s_2[n] \), the conditional p.d.f. of \( y'[n] \) is first maximized w.r.t. \( \tilde{h}_2 \) which results into the minimization of the following ML metric

\[
\Gamma'[n] = \ln(\det(A)) + \left( y'[n] - \tilde{h}_2 [s[n-1], s_2[n-1]s[n]] \right) A^{-1} \left( y'[n] - \tilde{h}_2 [s[n-1], s_2[n-1]s[n]] \right)^H.
\]

(15)

By using the fact that \( |s_i[u]|^2 = 1, i = 1, 2 \) and \( u = n, n-1 \), and after some algebra, the ML metric of (15) can be simplified as

\[
\Gamma'[n] = 2 \left| y'[n-1] - \tilde{h}_2 s[n-1] \right|^2 + 2 \left| y'[n] - \tilde{h}_2 s_2[n-1]s[n] \right|^2 + \text{Re} \left\{ s_1[n] \left( y'[n] - \tilde{h}_2 s_2[n-1]s[n] \right) \left( y'[n-1] - \tilde{h}_2 s[n-1] \right)^* \right\}.
\]

(16)

Differentiating (16) w.r.t. \( \tilde{h}_2 \) and setting the result to zero, leads to the value of \( \tilde{h}_2 \) which minimizes (16)

\[
\hat{\tilde{h}}_2 = \frac{y'[n-1]z_1[n] + y'[n]z_2[n]}{s[n-1]z_1[n] + s_2[n-1]s[n]z_2[n]}.
\]

(17)
where \( z_1[n] = 2s^*[n-1] + s_1^*[n]s_2^*[n-1]s_2^*[n] \) and \( z_2[n] = s_1[n]s_2^*[n-1] + 2s_2^*[n-1]s_2^*[n] \). Substituting the value of \( \tilde{h}_2 \) from (17) into (16), we get a differential decoder which completely avoids the knowledge of the channels of both users given as:

\[
\{ \hat{s}_1[n-1], \hat{s}_2[n-1], \hat{s}_1[n], \hat{s}_2[n] \} = \arg \min_{\{s_1[n-1],s_2[n-1],s_1[n],s_2[n]\} \in \Phi^4} \left\{ 2\left| y'[n-1] - \hat{h}_2s[n-1]\right|^2 + 2\left| y'[n] - \hat{h}_2s_2[n-1]s[n]\right|^2 \right. \\
+ \text{Re} \left\{ s_1[n] \left( y'[n] - \hat{h}_2s_2[n-1]s[n]\right) \left( y'[n-1] - \hat{h}_2s[n-1]\right)^* \right\} \right\}.
\]

(18)

It can be seen from (17) that the channel estimate is not defined for \( s_2[i] = s_1[i] \) for \( i \in \{n-1, n\} \). This means that we need to avoid this situation, i.e., we must choose the symbols of both users from different signal constellations. One possible solution is to choose the symbols of User 2 from a rotated signal constellation of User 1. Interestingly, this observation matches with the results obtained in [5], [6] for coherent decoder of two-user data over Gaussian MAC channels. In [6, Table I], optimized values of the rotation angles for different \( M \)-PSK constellations are obtained by maximizing the capacity of two-user Gaussian multiple access channel. In order to reduce the error in the channel estimate \( \hat{h}_2 \), we need to minimize the mean square error (MSE) in the channel estimate over all possible relative rotation angles between the two constellations. This observation leads to the following optimization problem:

\[
\text{minimize } \mathbb{E} \left\{ \left| \hat{h}_2 - \hat{h}_2 \right|^2 \right\}
\]

such that \( 0 < \theta < 2\pi \),

(19)

where expectation is performed upon the random variable (RV) \( \hat{h}_2 - \hat{h}_2 \). It can be seen from (19) that the optimization depends upon the p.d.f. of the RV \( \tilde{h}_2 - \hat{h}_2 \), which in turn depends upon the p.d.f. of the channel \( \tilde{h}_2 \) and its estimate \( \hat{h}_2 \). We have numerically plotted (19) the MSE of the channel estimate for different rotation angles and different \( M \)-PSK constellations at SNR=30 dB in Fig. 1 over uncorrelated Rayleigh fading channel. The values of MSE are obtained by numerically averaging the random variable \( \left| \tilde{h}_2 - \hat{h}_2 \right|^2 \) by using Monte Carlo simulations. We have noticed in the simulations that the optimized values of the rotation angle are different for Rayleigh and Ricean fading channels. Moreover, the optimized values of the rotation angle depend upon the variance of the channel. It can be seen from Fig. 1 that there exist multiple optimized values of the rotation angles for QPSK and 8-PSK for \( 0 < \theta < \pi \). However, we may
choose the lowest value of the optimized rotation angle. With this observation and from Fig. 1 we can modify the optimization problem of (19) to save the numerical computations as follows:

\[
\text{minimize } \mathbb{E} \left\{ \left| \hat{h}_2 - \tilde{h}_2 \right|^2 \right\}
\]

such that 0 < \theta < 2\pi/M. (20)

However, (20) does not guarantee a global optimum solution. Numerically optimized values of the rotation angles for different M-PSK constellations are listed in Table I. For simplicity, we have assumed that both users utilize the constellations of the same size. However, the optimization problem of (20) can also be solved for the case when both users utilize constellations of different sizes.

An alternative method for obtaining non-coherent decoders is to marginalize the p.d.f.s of the received data vector over the p.d.f.s of \(h_1\) and \(h_2\) and then try to derive an ML decoder of the data of both users. However, these decoders require the knowledge of the channel statistics like mean and variance for decoding the data. Whereas, the proposed heuristic differential decoder completely avoids the knowledge of the channels of both users and the proposed partial differential decoder completely avoids the knowledge of the channel of one user.

In the paper, we have taken an idealistic assumption that the transmissions of both users are perfectly synchronized, i.e., the time, phase, and frequency difference between the users are zero. The rotation angles are also calculated under this assumption. However, if the phase difference between the two users varies, then the users must know the phase and frequency of each other through the base-station, otherwise the relative phase rotation between the two constellations will drift from the desired rotation. This is a form of partial channel state information (CSI) exchange and improves the performance of the proposed scheme under the variable phase difference between the two users. If the users can exchange complete CSI, then it is possible to design a scheme which can perform better than the proposed scheme.

IV. Simulation Results

We have performed simulations with two users, BPSK constellation, Rayleigh fading channel with unity variance which remains constant over four consecutive time intervals, and \(10^4\) channel realizations. The data rate of the proposed scheme is 3/4 bits per channel use (BPCU) for each user. It is assumed that each user and the base-station is equipped with one antenna. The performance of the partially differential decoder of (9) assuming that the BS has perfect knowledge of \(\tilde{h}_2\) is plotted in Fig. 2. We have also plotted
the performance of the existing differential system [3] which transmits the data of each user in TDMA coordinated manner. The TDMA differential scheme utilizes 8-PSK constellation to achieve a data rate of 3/4 BPCU for each user. In the simulations, we have kept the total power transmitted in each time slot fixed for both schemes. Therefore, in the TDMA based scheme, each user transmits double power than that of each user in the proposed scheme. The SNR is set as the ratio of the average signal power transmitted in one time slot and average noise power for both schemes. It can be seen from Fig. 2 that the proposed partially differential decoder outperforms the same rate TDMA differential scheme for all SNRs. We have also shown the performance of the proposed heuristic differential decoder in (18) with optimized rotation angle. It can be seen from Fig. 2 that the proposed differential decoder with the optimized rotation angle performs better than the same rate TDMA based scheme taken from [3] at all SNRs given in the figure. For example, a SNR gain of approximately 3.5 dB can be obtained at SER=$10^{-2}$. Moreover, it must be noticed here that the TDMA based scheme needs a cooperation between the users to transmit the data in TDMA manner, whereas the proposed decoder works for the non-cooperative users who transmit over multiple access channels.

In Fig. 3, we have also plotted the SER versus SNR performance of the proposed heuristic differential decoder when two users utilize BPSK constellations rotated by different angles $\theta \in \{0, \pi/2, \pi/3, \pi/5, \pi/7\}$. The performance of the proposed heuristic decoder with the optimized rotation angle $\theta = 1.0739$ given in Table I for BPSK constellation is also shown in Fig. 3. It can be seen from Fig. 3 that the proposed differential decoder with $\theta = \pi/3$ performs almost similar to the numerically optimized rotation angle $\theta = 1.0739$ given in Table I. Therefore, it can be seen from Fig. 3 that the proposed scheme performs satisfactorily with the numerically obtained suboptimal value of the rotation angle.

V. Conclusions

We have proposed a differential decoder of two users transmitting over MAC channels. With the help of the proposed differential decoder, the receiver at the base-station can completely avoid the knowledge of the channel gains of both users. The proposed differential decoder is also able to achieve significant performance gain over the differential system where the two users need to transmit their data in a TDMA coordinated manner for facilitating differential decoding at the base-station.

REFERENCES


**TABLE I**

**Numerically optimized rotation angles for different M-PSK constellations**

<table>
<thead>
<tr>
<th>Constellation</th>
<th>Optimized rotation angle (rad.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPSK</td>
<td>1.0739</td>
</tr>
<tr>
<td>QPSK</td>
<td>0.7597</td>
</tr>
<tr>
<td>8-PSK</td>
<td>0.3513</td>
</tr>
</tbody>
</table>

**Fig. 1.** Numerical optimization of the rotation angle.
Fig. 2. Performance of the proposed differential decoder.

Fig. 3. SER versus SNR performance of the proposed differential decoder of (18) with BPSK constellations rotated by different angles.